Why under-constrained systems are not that bad

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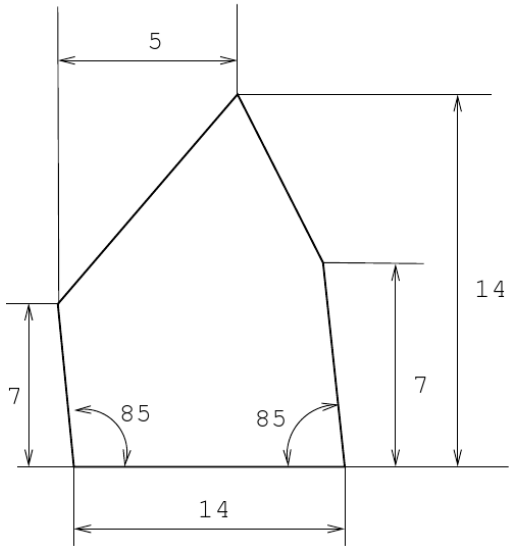
ADG '10: 8th Workshop on Automated Deduction in Geometry

Geometric Constraints Solving

- Input : a set of geometric constraints concerning geometric entities (sketch)
- solutions : figures satisfying the constraints (coordinates)



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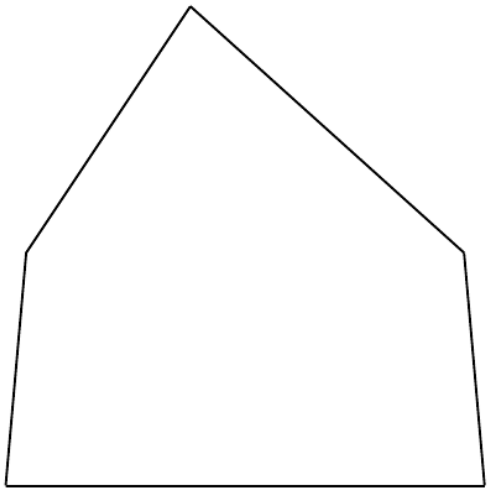

Geometric Constraints Solving

- Input : a set of geometric constraints concerning geometric entities (sketch)
- solutions : figures satisfying the constraints (coordinates)
- Application fields :
  - Computer-Aided Design
  - Computer-Aided Education
  - Molecular Chemistry
  - Robotics
  - ...

Solving methods

- algebraic methods : work on equations ;
  - symbolic : manipulate equations
  - numerical : iterative process
- geometric methods : reason on geometry
  - expert systems : explicit construction rules
  - graph-based methods : combinatorial methods
  - lack of generality
- general trend : decomposition

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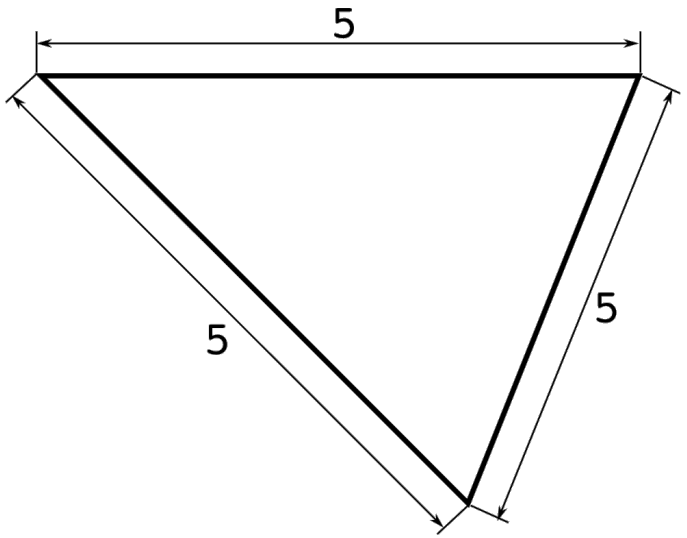
Constrainedness levels

- well-constrained : finite non-zero number of solutions (rigid)
- over-constrained : no solutions
- under-contrained : infinite number of solutions



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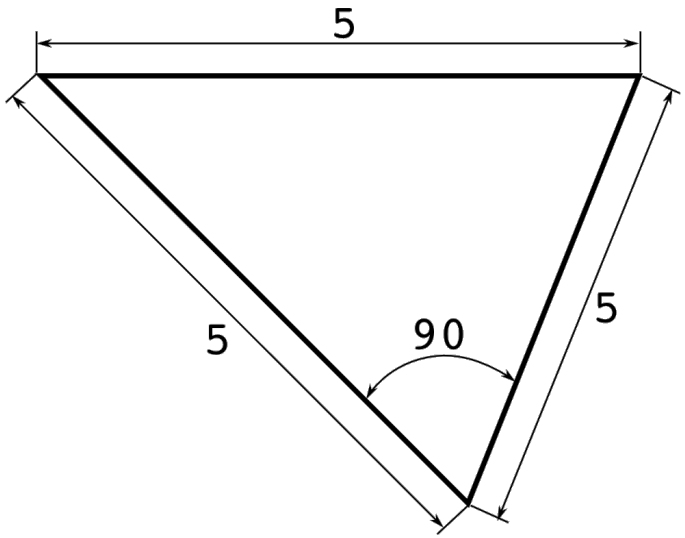

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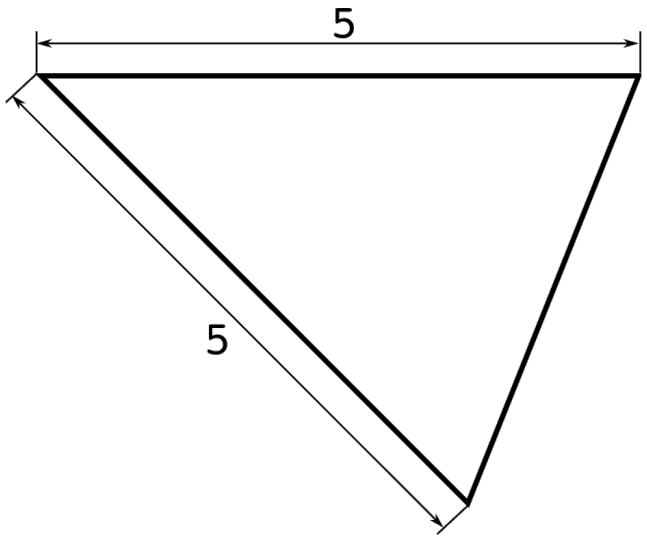

Under-constrained systems are an error

Under-constrainedness is an error case : it must be corrected by adding constraints to reach rigidity

Joan-Arinyo *et al.*, *Transforming an under-constrained geometric constraint problem into a well-constrained one*, 2003 : two issues :

- well-constrained completion (add constraints to an under-constrained GCS so that it becomes well-constrained)
- completion (add constraints in such a way that the new GCS can be solved by geometric constructions)

Zhang and Gao later (2006) added optimal well-constrained completion (add constraints so that the new GCS is well-constrained and the set of equations to solve simultaneously is of minimal size).

The literature contains a great deal of algorithms to complete an under-constrained system and make it rigid.

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Under-constrained systems are not that bad !

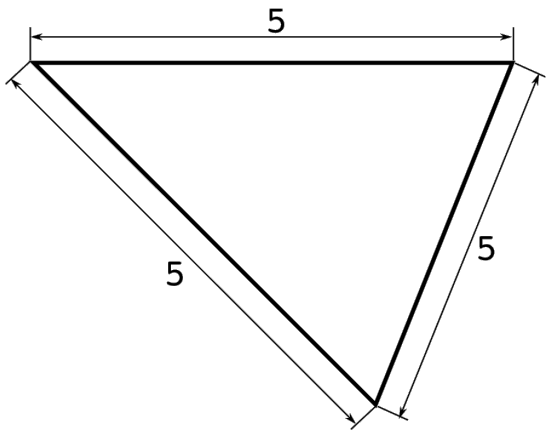
We should solve under-constrained systems the same way we solve rigid systems

- user intent (pair of scissors, desklamp, car, ...)
- path towards rigid systems
- needed for incremental methods
- application to dynamic geometry
- ...

Plan

I lied : rigid and yet under-constrained systems

- Rigid GCS



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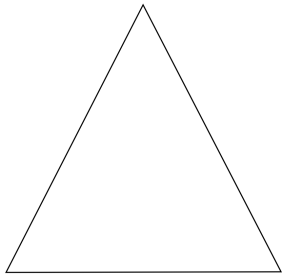

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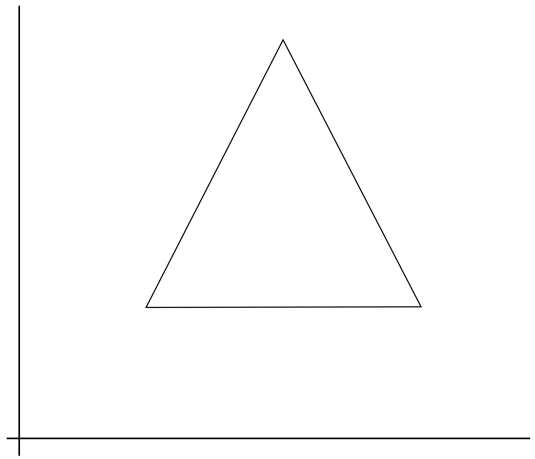

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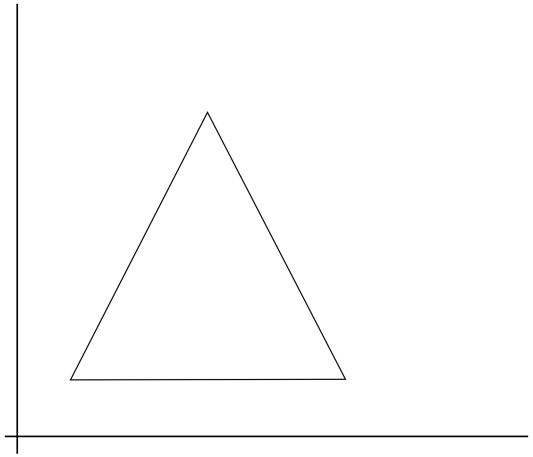
- Rigid GCS with an infinite number of solutions



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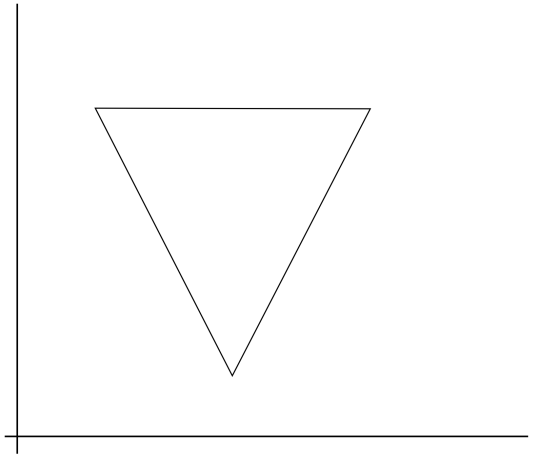

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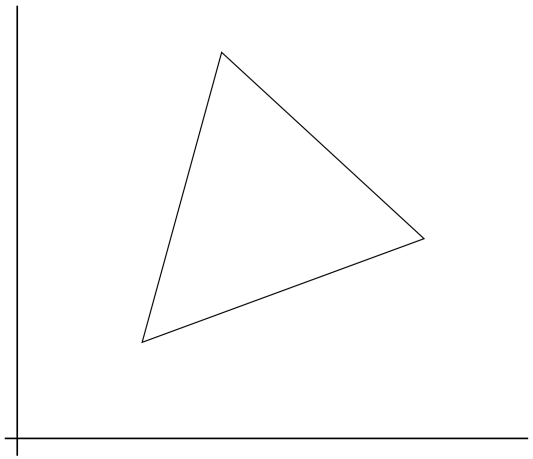

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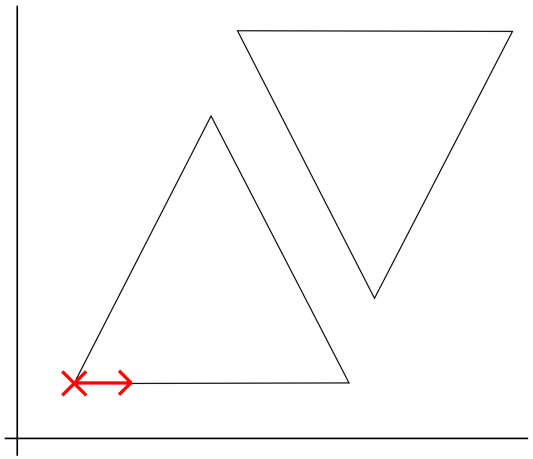
- Rigid GCS with an infinite number of solutions
- two particular solutions – reference
- apply rigid motions to generate the whole solution space



I lied : rigid and yet under-constrained systems

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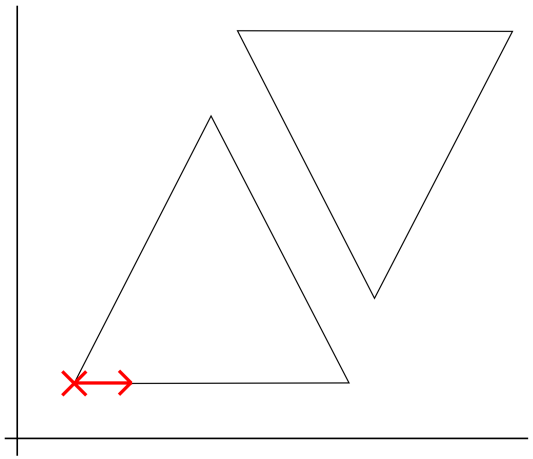
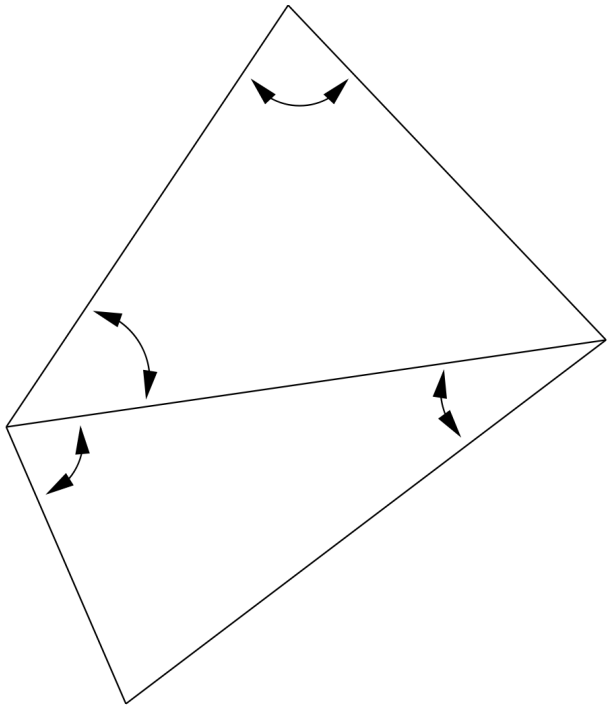
- two particular solutions – reference
- apply rigid motions to generate the whole solution space
- Well-constrained *modulo* rigid motions

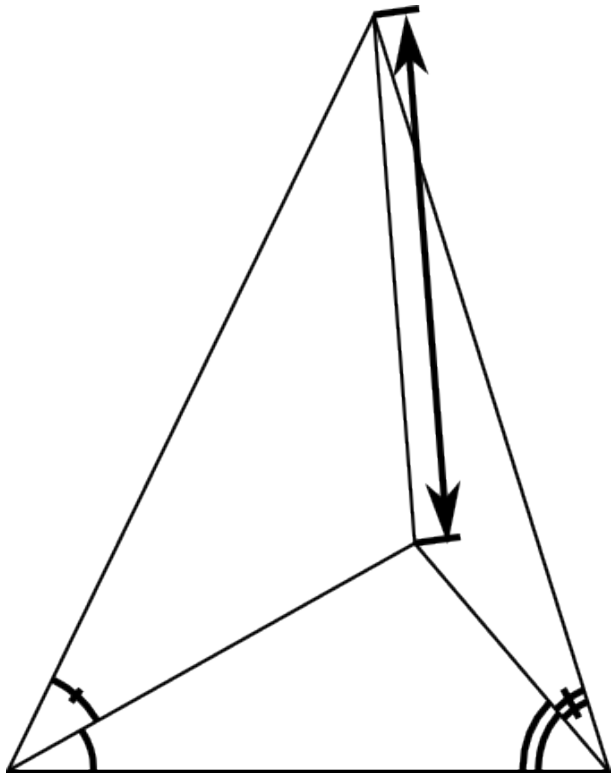


Multi-group approach

- transformations groups :
  - translations
  - rotations
  - scalings
  - and their compositions
- act globally
- reference for a transformation group



Useful for decomposition



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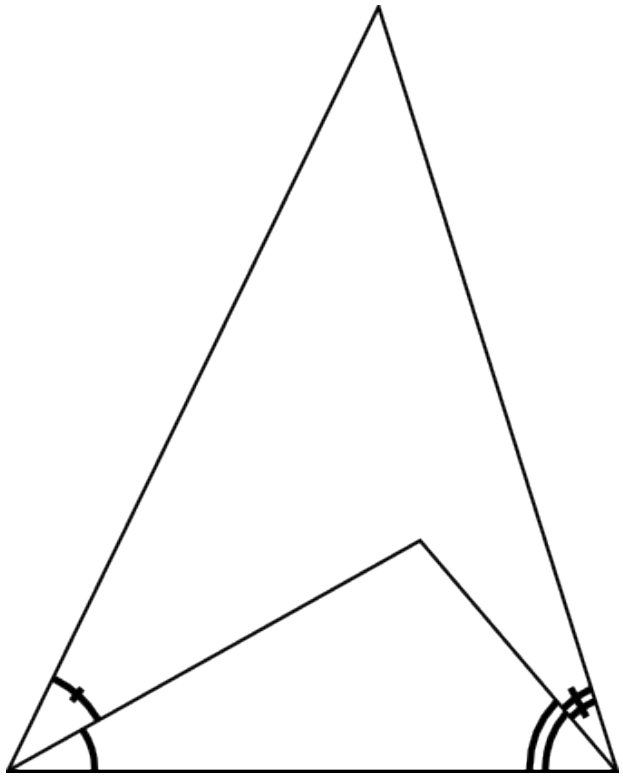

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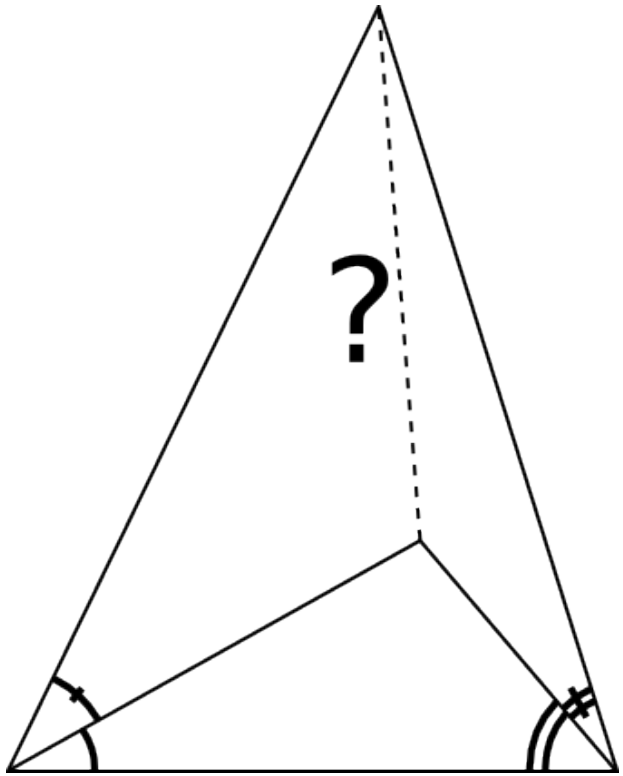
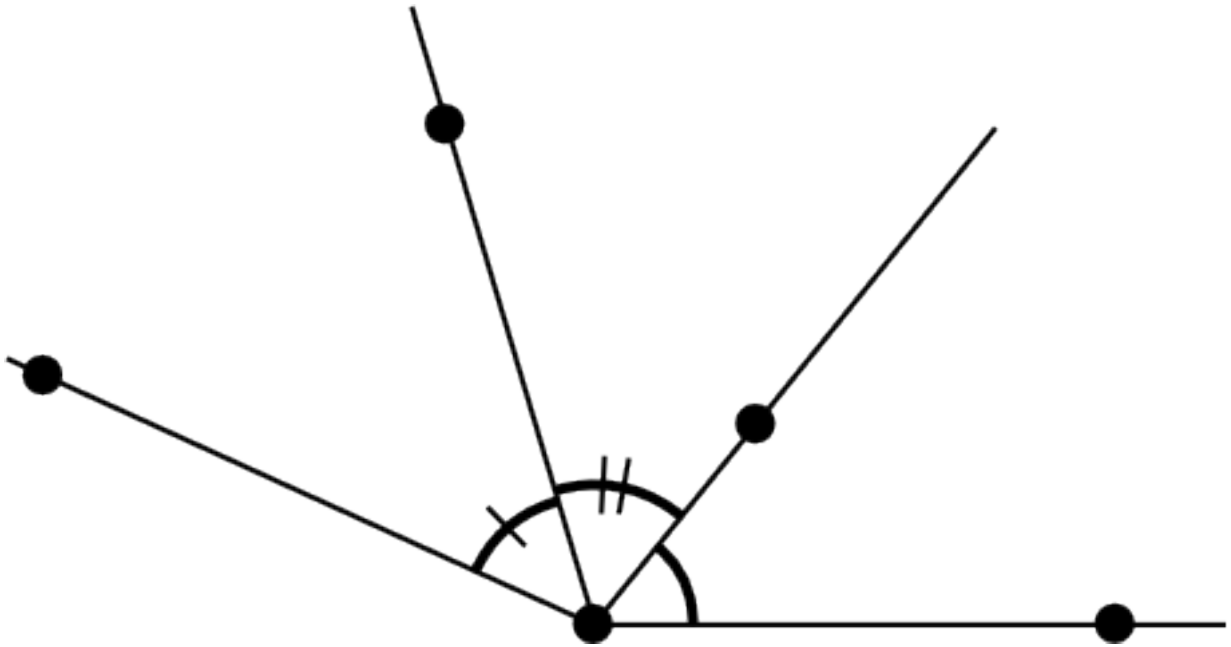


Schreck and Mathis, *Geometrical constraint system decomposition : a multi-group approach*, 2006

Useful for decomposition [2]

van der Meiden and Bronsvoort, *A non-rigid cluster rewriting approach to solve systems of 3D geometric constraints*, 2010
Three kinds of clusters :

1. classical rigid clusters
2. scalable clusters (well-constrained *mod* similarities)
3. radial clusters



W-decomposition

- Witness interrogation method : Michelucci and Foufou, ADG '06
- allows to identify the maximal rigid subsystems (MRS)
- search for the MRSs of under-constrained subsystems of a rigid system
- remove a constraint, identify MRSs, recurse

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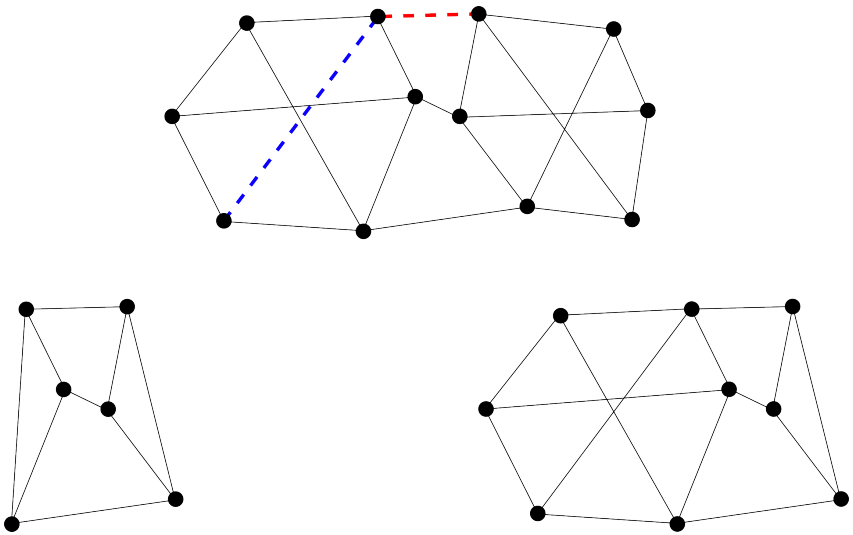

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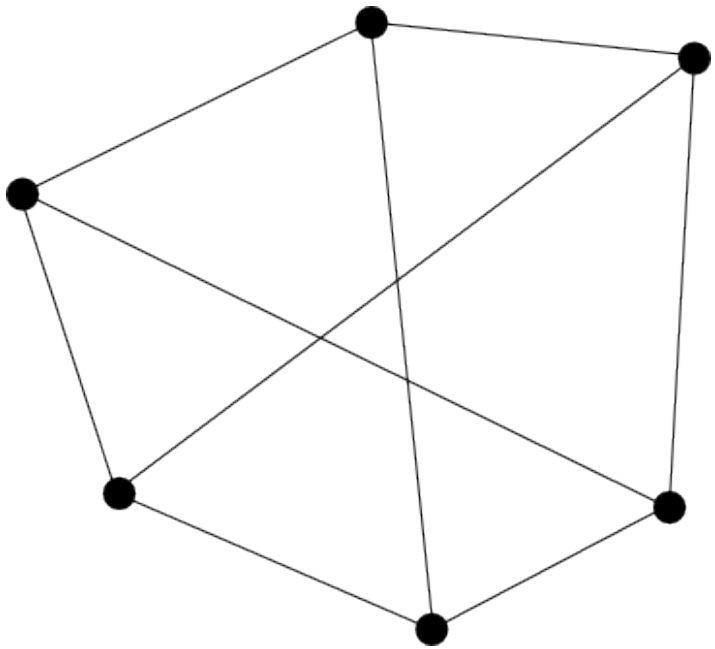
Relax...

Fabre and Schreck, *Combining symbolic and numerical solvers to simplify indecomposable systems solving*, 2008

- system you cannot decompose
- but you can solve one of its under-constrained subsystems
- add parameterized constraints
- numerical iterations



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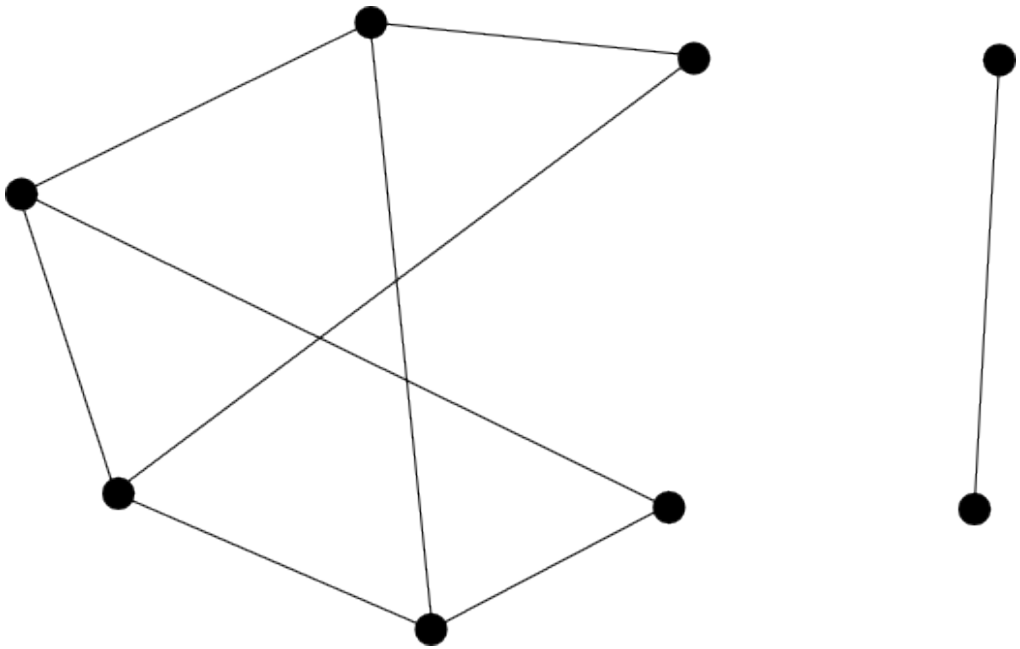

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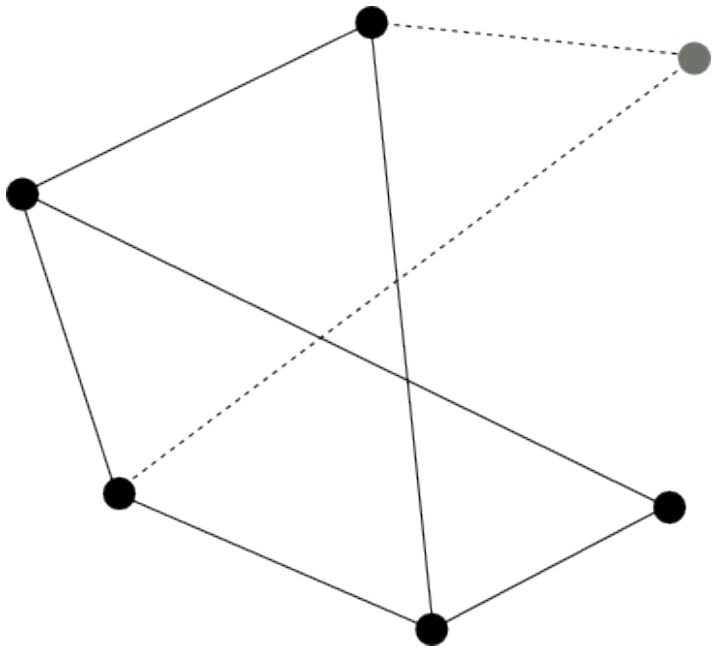

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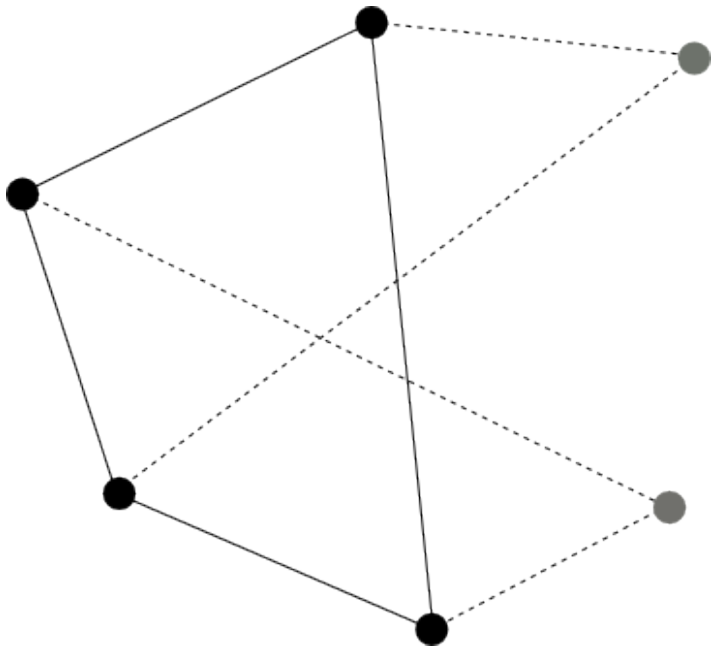

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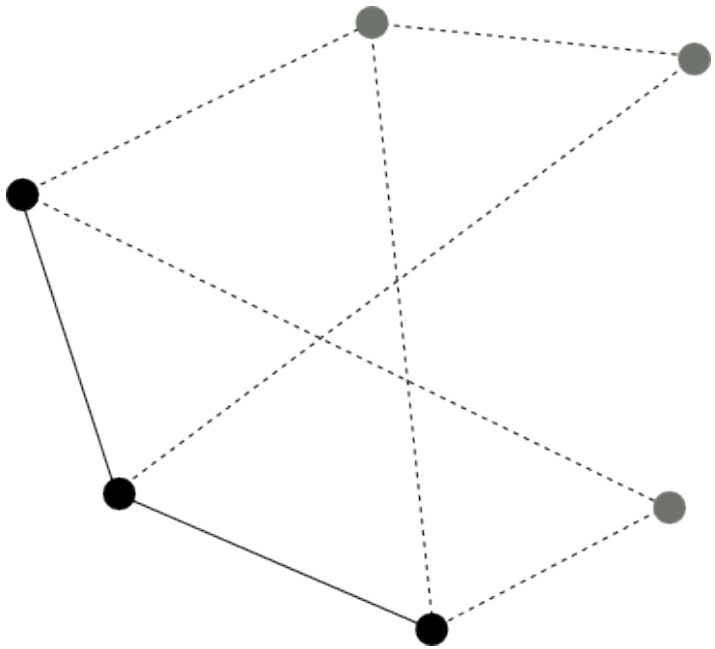

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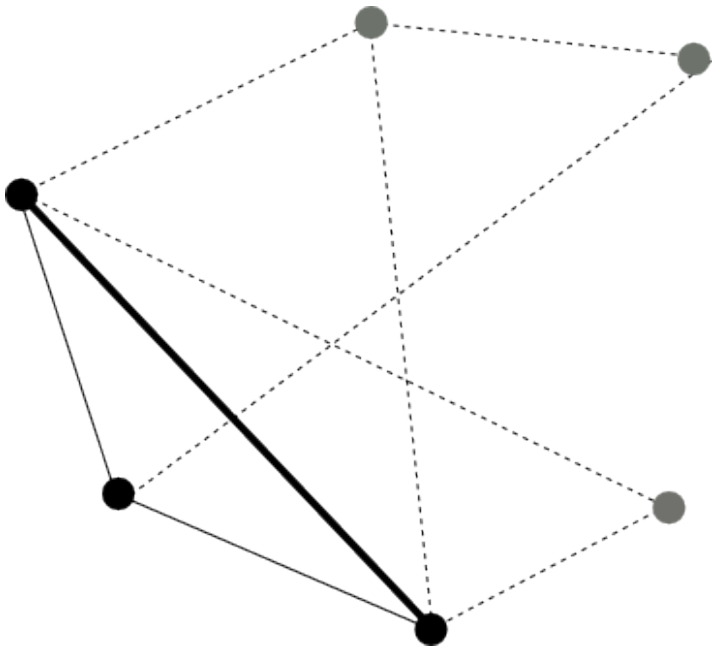

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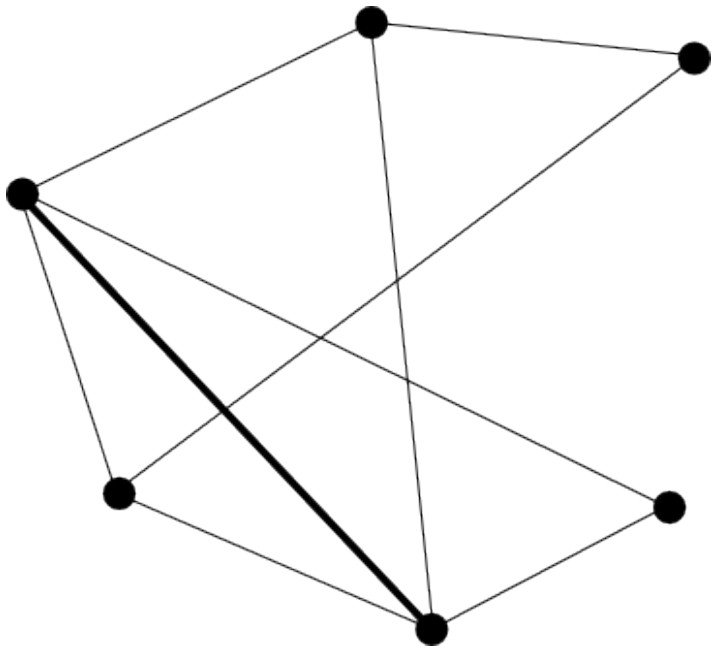

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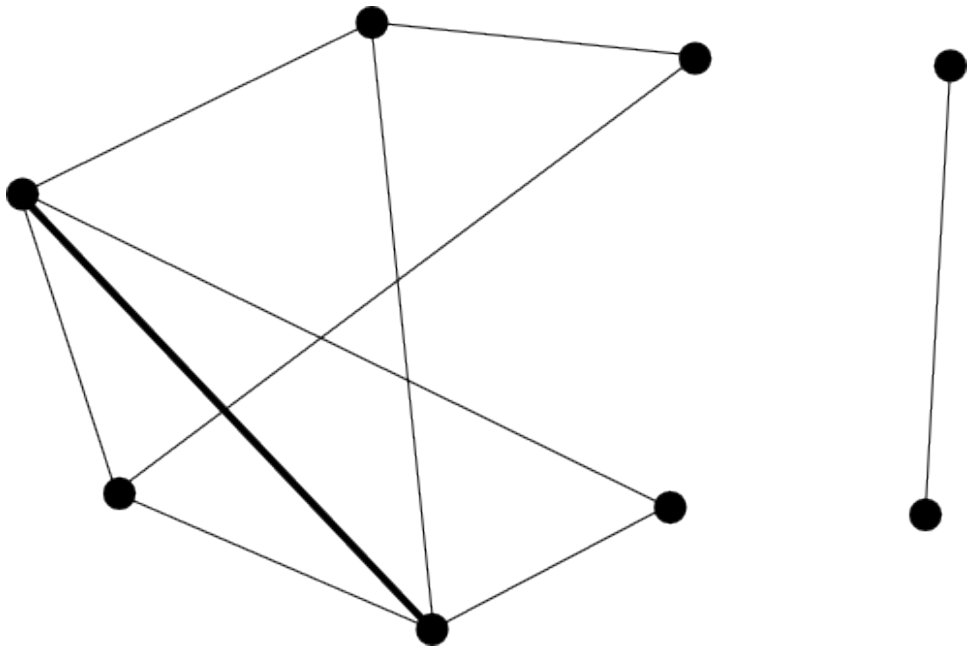
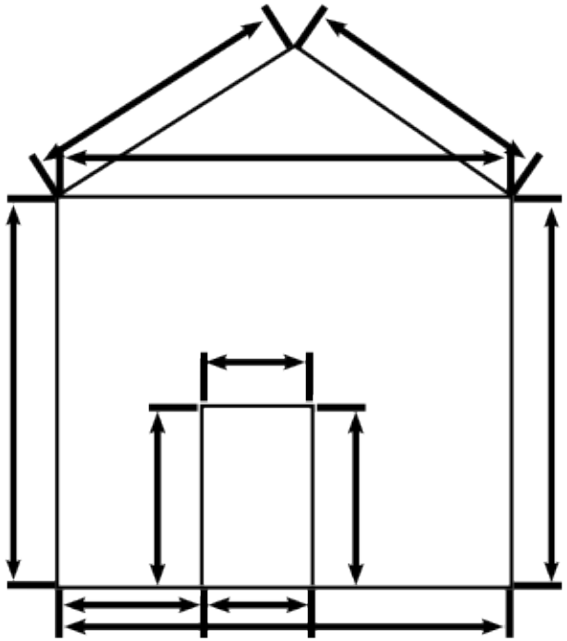


Non-expert users and under-constrainedness



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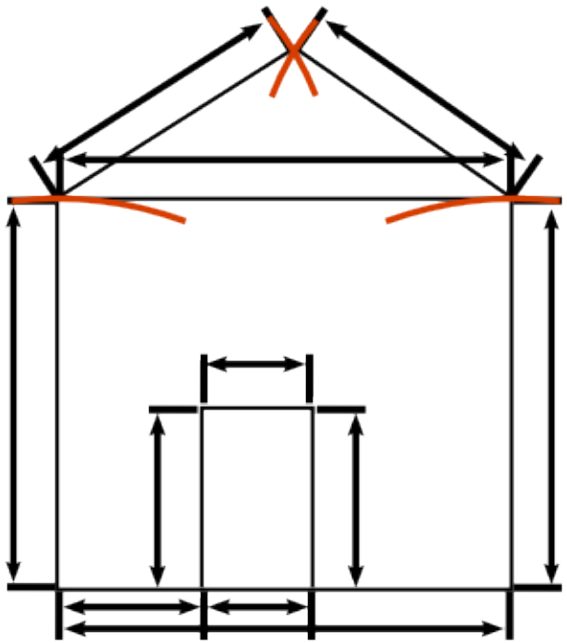

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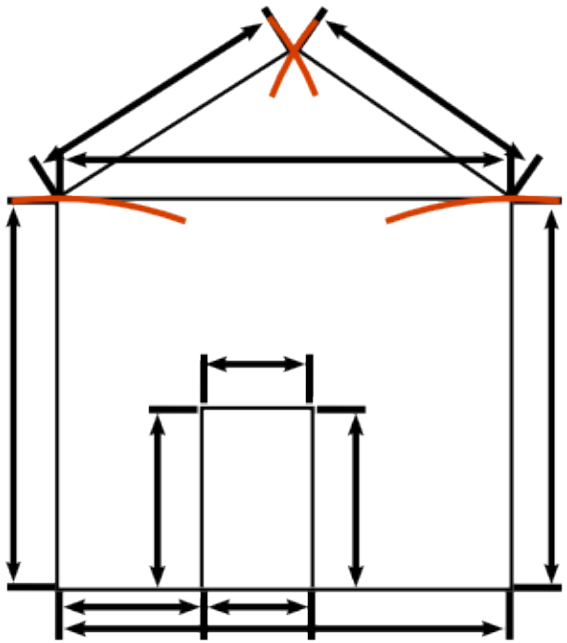

Under-constrained : the users considered implicit constraints

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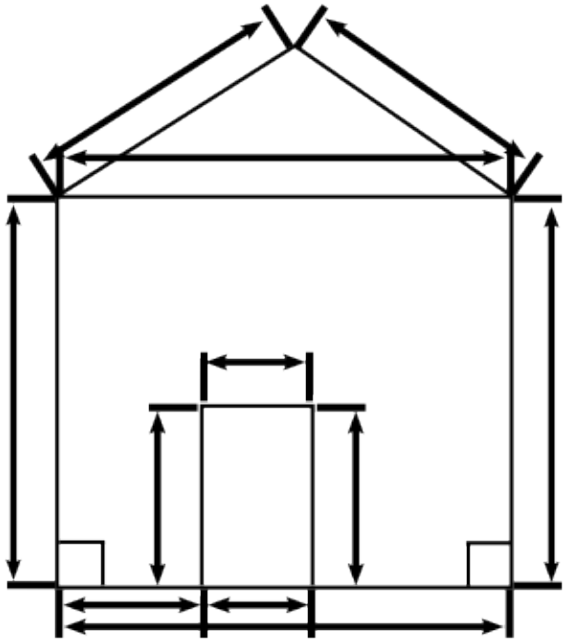

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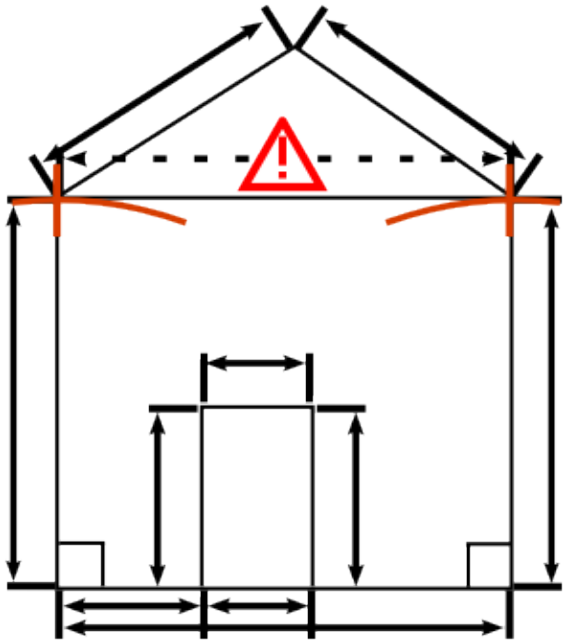

Over-constrained : distance cannot be generically satisfied

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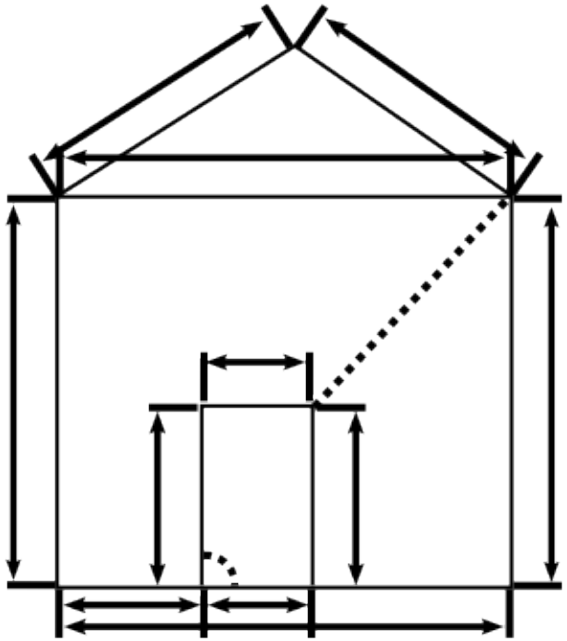
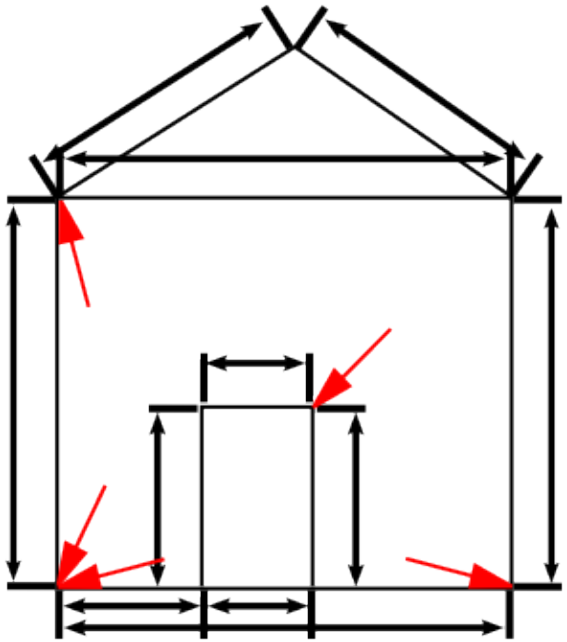


Parameterization

The other approach : tell the user which geometric elements should be anchored.

- compute a parameterization
- compute appropriate values for the parameters
- compute solution figures

Enables trial and error modeling.



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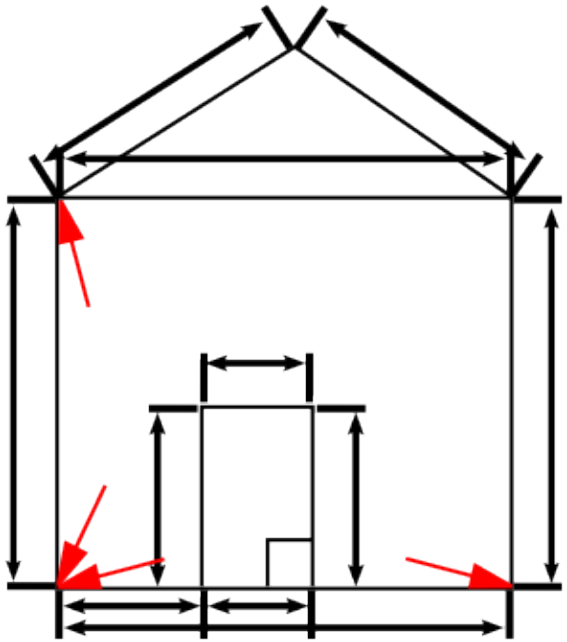
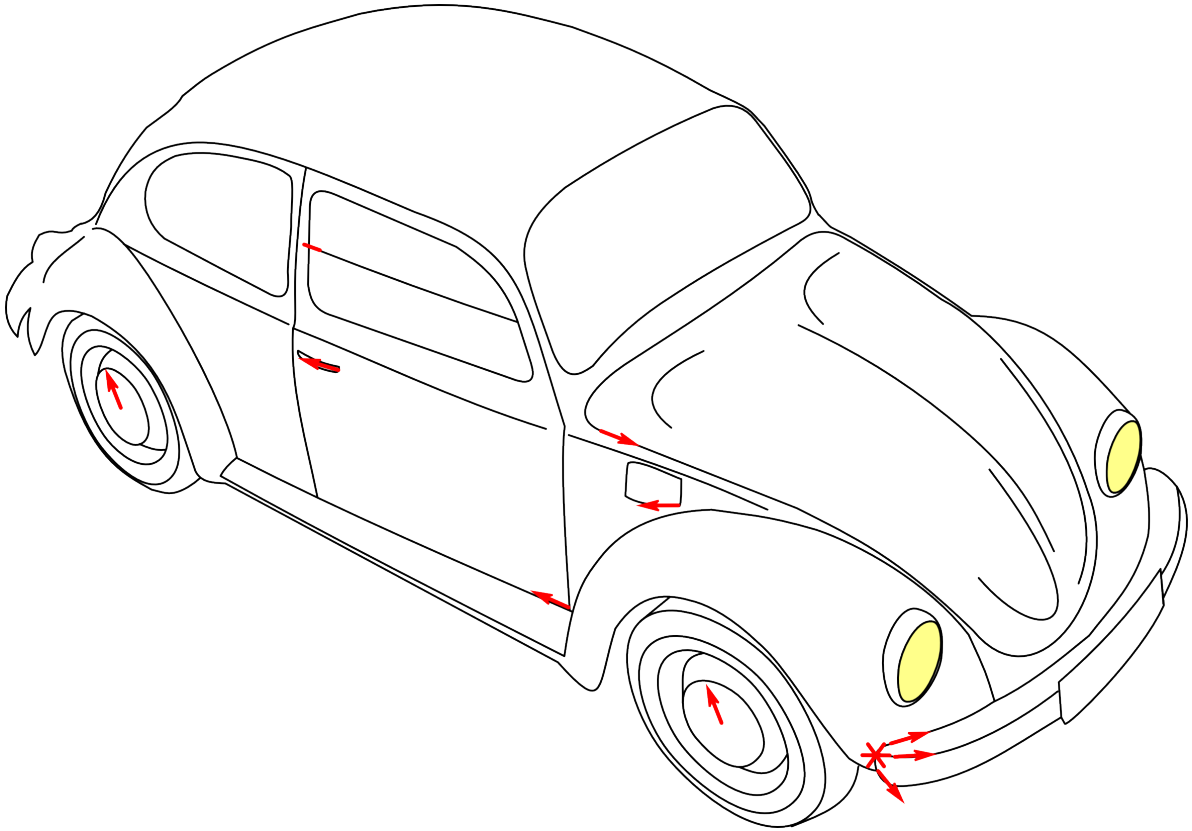


Accessible user interfaces



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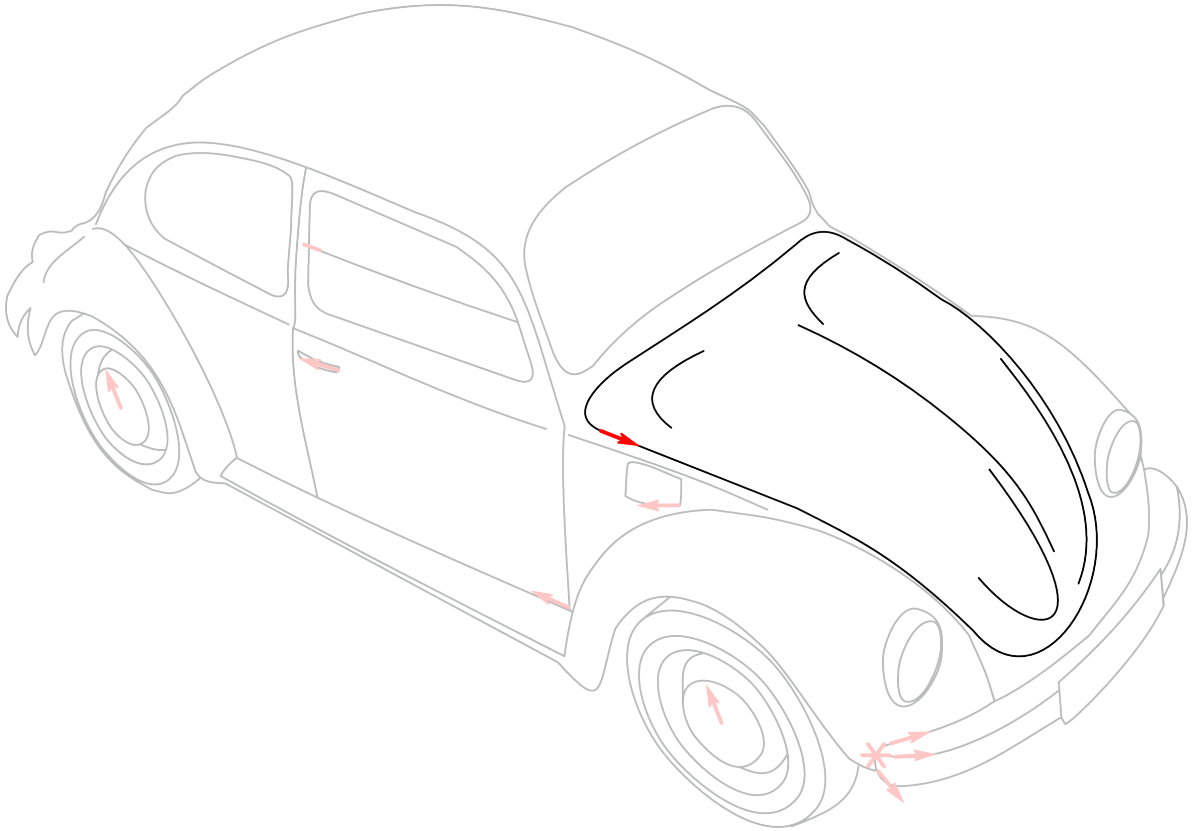

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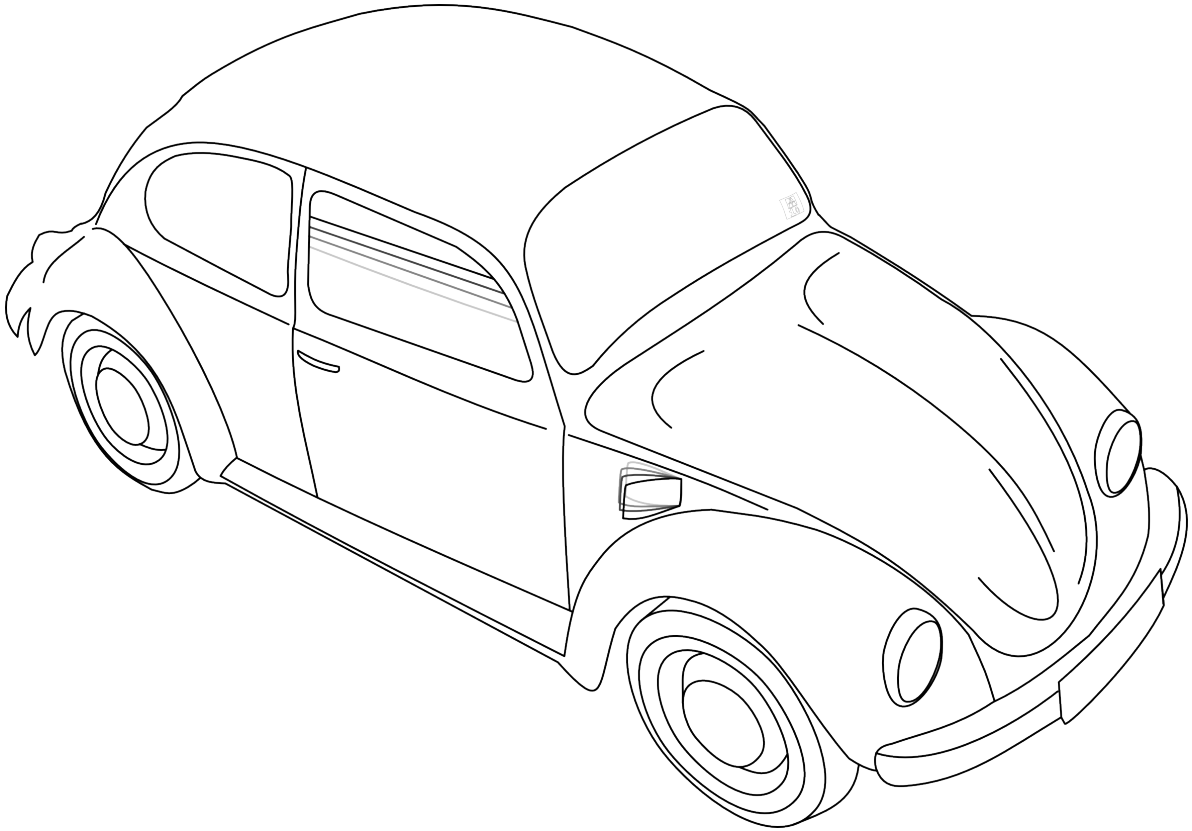

Conclusion

Under-constrained geometric constraint systems should not be considered as design errors

- using under-constrained systems helps solving rigid systems
- rigid systems and under-constrained systems should be completed the same way : by anchoring them in the plane/space
- leads to accessible constraint-based design softwares

Thanks for your attention